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**On the Impossibility of Reducing the Surplus Approach
to a Neo-Classical “Special Case”**

A criticism of Hahn in a “Solowian” context

Emiliano Brancaccio

We propose a new criticism of Hahn’s attempt to prove that the Surplus approach constitutes no more than a “special case” of the Neo-Classical model of intertemporal general equilibrium (Hahn 1982). In particular, we show that Hahn’s “special case” is vitiated by the paradox of determining the past as a function of the future. In order to make the communication between schools of thought easier, we present our criticism of Hahn within a mathematical framework drawn from the well-known Solow growth model (1956), which is particularly familiar to mainstream economists.

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Address for correspondence

Emiliano Brancaccio
Research fellow of Political Economy
DASES – Università del Sannio
Via delle Puglie 82
82100 Benevento, Italy
emiliano.brancaccio@unisannio.it

1. Introduction¹

Known also as the “Theory of production” or “Neo-Ricardian theory”, the “Surplus approach” originated in the work of the Classical economists and Marx and developed more recently around the work of Piero Sraffa (1960) and his successors.² In this paper we propose a comparison between the Surplus approach and Neo-Classical theory within a common mathematical framework drawn from the well-known Solow growth model (1956). In particular we develop a new criticism of Hahn’s notorious attempt (1982) to prove that the Surplus theory constitutes no more than a “special case” of the Neo-Classical model of intertemporal general equilibrium. As we shall see, it is possible to demonstrate that Hahn’s “special case” cannot be accepted because it is vitiated by the paradox of determining the past as a function of the future. Many aspects of the comparison between the two theories examined cannot of course be adequately developed within the Solowian framework adopted here and will necessarily be passed over.³ At the same time, however, the familiarity of the formal structure adopted here should help mainstream economists to understand the basic reasons why the Surplus approach is logically incompatible and alternative to Neo-Classical theory.

2. The Surplus approach in Solowian dress

Let us start by ascertaining whether the basics of the Surplus approach can actually be presented within a formal structure drawn from Solow (1956). In particular, we shall now address the question of whether the key ideas of Surplus theorists can be also described within the framework of a model of development and distribution with a single good and flexible coefficients based on a Solowian mathematical structure. To this end, we shall present the stylised description of a capitalistic system with no foreign trade producing a single good by means of labour and the good itself. Let us begin by describing the technology of the system. Let K be the physical amount of the good available as capital and therefore used as production input, L the quantity of homogeneous labour employed and X the quantity of the good produced. We thus obtain the following production function:

$$X = F(K, L)$$

On the assumption that the production function has constant returns to scale, it is possible to state that:

$$\alpha F(K, L) = F(\alpha K, \alpha L) \quad \forall \alpha \in R^+$$

By positing $\alpha = 1/L$, the function can be rewritten as follows:

$$x = f(k)$$

where $x = f(k) = X/L$ represents output per unit of labour and $k = K/L$ capital per unit of labour, i.e. the technique of production adopted. It should be noted that once k has been determined, the ratio between K and X is also known because $K/X = k/f(k)$.

Let W be the monetary wage, r the rate of profit, P the level of monetary prices of the good produced and $Y = PX$ the monetary value of the output and hence income. The value of the output achieved will be equal to:

$$Y = PX = WL + (1+r)PK$$

This expression can be used in various ways. In our case, division by P and L gives us the distribution of the physical output between wages and total gross profits prior to replenishment of capital for every given unit of labour:

$$(1) \quad f(k) = \frac{W}{P} + (1+r)k$$

Let us now go on to examine the coefficient k , which can be regarded as fixed or flexible depending on the assumptions as regards technology. With a view to facilitating comparison with the Neo-Classical analysis, it is assumed here that the production function is continuous and differentiable and that it meets the following conditions:

$$f(0) = 0, \quad f'(k) > 0, \quad f''(k) < 0$$

As k can therefore assume infinite values, a problem arises as regards choice of the optimal technique. This can be determined once the rate of profit is given. Firms will in fact tend to opt for the technique k that maximises the difference:

$$\max \quad f(k) - (1+r)k$$

which indicates output net of the amount of the rate of profit, all in terms of units of labour. Maximisation means that:

$$(2) \quad f'(k) = 1+r$$

In other words, firms will opt for the particular technical combination k of capital and labour ensuring fulfilment of the maximum condition described by equation (2).

Equations (1) and (2) are formally identical to those contained in the Solowian growth models and will therefore be familiar to Neo-Classical

economists. They will also appear very far removed from the typical characteristics of the Surplus approach at first sight. Suffice it to consider the assumption that the production function is continuous and hence that k can assume infinite values. This hypothesis forms no part of the theoretical tradition of the Surplus approach and has indeed been criticized quite frequently by Sraffian economists. It should, however, be pointed out that these distinguishing elements are not crucial in the sense that the specific functional forms adopted are less important for our understanding of the structure of a theory than the general relations between the exogenous and endogenous variables.⁴ On given assumptions, equations (1) and (2) can therefore provide a schematic representation of some basic characteristics of the Surplus approach. The system to be addressed thus consists of two equations with four unknowns: r , k , W and P . On the assumption that the rate of profit r is exogenous, equation (2) makes it possible to determine the optimal technology k that firms will choose. With r given and k known, equation (1) will make it possible to determine the real wage W/P . Finally, for every given monetary wage W , it will also be possible to obtain the corresponding level of monetary prices P . It should be noted that the entire process makes sense on the assumption that once the technology k has been determined, the ratio between the quantities K and L can adapt to it.

The particular sequence described above reveals that the Surplus approach is based on the idea that endowments of production inputs are not exogenous but adapt to any given distribution of income once the corresponding technique of production is known. This possibility of adjustment is based on the hypothesis that labour can be considered abundant, the means of production are themselves produced, and their degree of utilization is highly flexible. This means that in the Surplus approach distribution and prices can be determined without reference to the “scarcity” of production inputs. On the contrary, as we shall see, Neo-Classical theory always takes the initial endowments of production inputs as its exogenous variables. Prices and distributive variables are then determined endogenously and should guarantee that the given endowments will be absorbed by their respective demand. Which is why in the Neo-Classical theory prices and distributive variables are considered indicators of the scarcity of factors of production in relation to demand.

3. A new criticism of Hahn

Two antagonistic views of scarcity and prices - and of the consequent causal relationships between variables - thus seem to establish the existence of an unbridgeable gap between Neo-Classical theory and the Surplus approach. However, not everybody would agree that the two theories are logically incompatible. In this connection, we can profitably focus attention on Frank Hahn's well-known article *The Neo-Ricardians* (1982). Regarded by many as one of the most influential contributions to 20th-century debate on economic theory,⁵ it constitutes the most authoritative attempt to put a definitive end to the well-

known dispute between the two schools of thought at Cambridge on the theory of capital,⁶ aiming explicitly at securing a clear victory for advocates of the Neo-Classical approach. Hahn maintains in his article that Sraffa's criticism of the theory of capital fails to identify any logical flaw in the "short-period" version of the Neo-Classical analysis based on the model of intertemporal general equilibrium. Furthermore, Hahn endeavours to prove that the Surplus approach inspired by Sraffa's views constitutes no more than a largely insignificant "special case" of the Neo-Classical model of intertemporal equilibrium.⁷

Hahn's approach and conclusions still appear to enjoy widespread acceptance in academic circles⁸ and were explicitly reasserted by the author himself quite recently.⁹ It has, however, been demonstrated that his analysis is vitiated by a series of serious errors of logic and method. It has been argued that the goal of reducing the Surplus approach to a "special case" of the Neo-Classical model is flawed from the very outset because these are theoretical constructions based on two sets of hypotheses that are completely different and mutually incompatible. It has been suggested that the short-period versions of Neo-Classical analysis are also vulnerable to Sraffian criticism of the theory of the capital, contrary to Hahn's claims. It has been said that the Neo-Classical theory of intertemporal equilibrium could be considered a "special case" of the Sraffian theory of the prices.¹⁰ Furthermore, Garegnani (2003) noted that in building his "special case" Hahn considers «physical compositions of capital endowment the economy moves away from, as equivalent to [...] physical compositions the economy tends to».¹¹ Though rigorously argued and in many respects unanswerable, these criticisms appear to have been to some extent ignored in mainstream economics. To this respect, there seems to be a problem of non-communication between schools of thought.

In this paper we cannot consider all these objections to Hahn. In particular, we shall not go into the important criticism of Hahn's attempt to rehabilitate the Neo-Classical theory of capital. In what follows we shall develop only the criticism of Garegnani (2003) by showing that Hahn interprets Sraffians in an odd way, as if they intended to reverse the time axis and determine the past as a function of the future. In comparison with Garegnani, our innovation is that we consider in depth the final phases of the operation carried out by Hahn, which consist, as we shall see, in linking the rate of profit to an exogenous rate of growth. Moreover, in order to make the communication easier, we present our criticism within a one-good Solowian mathematical framework, which is particularly familiar and accessible to mainstream economists.

4. The Neo-Classical "general case"

Let us examine how Hahn endeavours to interpret Surplus theory as no more than a "special case" of the "general" Neo-Classical analysis. As we shall see, Hahn rejects a priori the typical Sraffian idea that a distributive variable can be regarded as exogenous and above all that it can be considered independent of the

endowments of productive factors.

Faced with equations (1) and (2) in their Sraffian interpretation, Hahn would maintain that they in no way represent the basis of an alternative system but correspond instead to the equations of a typically Neo-Classical model whose only peculiarity lies in the absence of the equation needed to determine r endogenously. He therefore sets out in search of the “missing equation” (Hahn 1982, section 5) with a view to constructing a Neo-Classical model defined as “general”, i.e. capable of encompassing what he regards as the Sraffian “special case” and hence of determining the rate of profit and all the other distributive variables endogenously. To this end, he constructs a system of intertemporal general equilibrium representing an economy that has only two periods. As presented in the original article, the system produces two goods. In what follows Hahn’s arguments are faithfully reported but adapted to a Solowian context with only one good. In this particular sphere, the equation corresponding to the “equilibrium condition for producers” contained in Hahn’s “general” Neo-Classical model can be rewritten as follows:¹²

$$P'_t X_t = W'_t L_t + P'_{t-1} K_t$$

It should be noted that the prices P' are not expressed in money here but in terms of the good at time t :

$$P'_t = \frac{P_t}{P_t}, \quad P'_{t-1} = \frac{P_{t-1}(1+i)}{P_t}, \quad W'_t = \frac{W_t}{P_t}$$

Hahn’s equilibrium condition initially appears to bear no resemblance to the equation (1) adopted here. However, if all of his relative prices are multiplied by the monetary price P_t , the condition becomes:

$$P_t X_t = W_t L_t + (1+i)P_{t-1} K_t$$

Furthermore, given the typical arbitrage condition (Solow 1956):

$$1 + r = \frac{P_{t-1}(1+i)}{P_t}$$

we have:

$$P_t X_t = W_t L_t + (1+r)P_t K_t$$

and equation (1) is easily obtained by dividing the whole by $P_t L_t$. We have thus shown that the condition of equilibrium contained in Hahn’s (1982) “general” model corresponds exactly to equation (1). Moreover, the assumption that it is possible to choose from a variety of technical combinations k allows us to add

equation (2) to the model.

The “general” Neo-Classical case is, however, not yet complete. It is in fact still necessary to find what Hahn describes as the “missing” equation in order to close the system. To this end, the equation of equilibrium between production and demand for goods can be introduced into the analysis. Let C and I be the aggregate monetary values of consumption and investment respectively. We can now write:

$$Y = C + I$$

Let us also define aggregate saving as $S = Y - C = sY$, where s is the average propensity to save of the population. Remembering that $Y = PX$, assuming that $I = (1+g)PK$ and dividing the whole by PL , we can now rewrite the equation as follows:

$$(3) \quad sf(k) = (1 + g)k$$

The model (1), (2) and (3) can be understood as the single-good version of Hahn’s Neo-Classical “general case”. The system is made up of three equations with five unknowns - namely r , W , P , g and k - with s representing a parameter determined by the habits of the population and consequently assumed as given. The solution is as follows. As the endowments of the inputs of capital and labour K and L available at the beginning of every period are exogenous, their ratio k also proves to be determined. The system is therefore complete with (2) determining r , (1) determining W/P and (3) determining g .¹³ Finally, on the typically Neo-Classical assumption that P is given by quantitative theory, the monetary wage W of equilibrium can also be obtained from (1).

The “general” model described here thus takes the endowments of factors given at the beginning of the period as its starting point and determines the distributive variables endogenously. It should also be noted that in this model the physical magnitudes do not necessarily grow at the same rate. For these reasons, the model can be made to correspond to an equilibrium described as “non-stationary” within the framework of Solow’s analysis and usually termed “short-period” in the context of the intertemporal models.¹⁴ Anyway, it is important to specify that a switch to an equilibrium of stationary growth would not change the basic features of the Neo-Classical “general” model. In particular, even in a stationary growth distributive variables would be dependent on the endowments of productive factors.¹⁵

5. The Sraffian “special case”

Hahn regards the model described above as showing that the Neo-Classical theory is capable not only of encompassing equations (1) and (2) but also of determining the rate of profit r , which Sraffian economists instead prefer to regard as

exogenous. This result cannot, however, be considered satisfactory as yet for Hahn's purposes. If the Neo-Classical model is to have the character of a "general case", he must in fact show that it can determine r in compliance with a key Sraffian assumption of the Surplus approach, according to which the distribution of income can be determined independently of any reference whatsoever to the technique of production and the endowments of production inputs. This is not in fact true of the "general" model as described so far, since r turns out to be a function of k and hence ultimately dependent on the technique of production and the initial endowments of K and L . Hahn accounts for this, however, on the grounds that the Surplus approach is only a "special case" and that conclusions compatible can be obtained simply by imposing suitable constraints on the "general" Neo-Classical model (Hahn 1982, section 5). To this end, he first suggests the introduction of a particular assumption as regards accumulation decisions. Usually referred to as the "Classical saving hypothesis",¹⁶ this is in fact used by some theorists of the Surplus approach. In its extreme version, the hypothesis rests on the idea that the society is divided into two social classes, namely capitalists and workers, and that while the former save all of their profits, the latter consume all of their wages. This means that aggregate saving corresponds to total profits:

$$S = (1 + r)PK$$

Secondly, Hahn's solution involves recourse to equation (3) of macroeconomic equilibrium also for the Sraffian "special case". Given the Classical saving hypothesis and the customary division by P and L , the macroeconomic equilibrium represented by (3) becomes:

$$(1 + r)k = (1 + g)k$$

from which it is possible through simplification to obtain:

$$(3') \quad r = g$$

It would in fact appear somewhat imprecise to describe a system to be closed with an equation of equilibrium between income and aggregate expenditure as "Sraffian". On this issue, there has been a long debate among the representatives of the Surplus approach.¹⁷ However, Hahn does not take this discussion into account, and frequently appears to believe that the connection between the macroeconomic equation, rate of profit and production prices can be associated with "Sraffians" or "Neo-Ricardians" in the broad sense. His model of the Sraffian "special case" is therefore made up of equations (1), (2) and (3') and the five unknowns r , W , P , g and k . At this point, if it is assumed this time that g is exogenous and k endogenous, a new resolute sequence is obtained. Equation (3') determines the rate of profit r exclusively on the basis on the growth rate g . Therefore, once r is known, the technical combination k is determined by (2). And it should be made clear that once k has been determined, the factorial endowments

K and L will have to prove compatible with it. Finally, with P given once again by quantitative theory, it is also possible to obtain W from (1).

Hahn therefore believes he has proved that it is possible, by applying suitable constraints, to determine a “special case” from the “general” Neo-Classical model capable, in his view, of satisfying all the conditions of the Surplus approach inspired to the works of Sraffa, and especially the prerequisite of an r independent of the production technique and initial endowments of production inputs.¹⁸ Hahn insists, however, on adding that this “special case” is insignificant because the result at which it arrives can only be regarded as consistent only when the ratio of the initial endowments of K and L happen to coincide with the term k determined as a function of the rate of profit and therefore ultimately of the exogenous rate of growth. These elements can, however, coincide only by chance, since there is no logical reason why they should necessarily do so. This is why Hahn ironically concludes that if this coincidence does not come about, it means that Sraffa is “out of luck” together with all the theorists of the Surplus approach.¹⁹

6. The paradox of the past as a function of the future

We have already mentioned the logical error identified by Garegnani (2003) in Hahn’s argument. This error derives from the fact that Hahn considers the physical compositions of capital endowment the economy moves away from as equivalent to physical compositions the economy tends to. In other words, Hahn proposes an odd interpretation of Sraffians in which the time axis is reversed and the past is confused with the future. Our purpose here is to resume and develop Garegnani’s objection in a Solowian mathematical framework. In this particular context, we intend to show that the so-called “special case” does not comply with the Sraffian canons because the rate of profit is not determined independently of the technique of production and endowments of production inputs. This becomes clear on examination of the solution envisaged by this “special case”, which takes the assumption that the rate g of capital accumulation is exogenous as its starting point. In the Neo-Classical sphere, however, accumulation depends on the savings available. Therefore, if g is exogenously determined, a certain amount of savings should be implicitly required in order to generate that particular rate of accumulation. If a Classical saving hypothesis is then assumed, as it is in Hahn’s argument, attainment of the required amount of saving will in turn necessitate a certain rate of profit. This means that g determines saving unequivocally and therefore determines r on the basis of equation (3’). The point is that, as derived from equation (2), r in turn determines the optimal ratio k at which firms will employ the factors of capital and labour. If k is already determined, however, the problem arises of its compatibility with the endowments of K and L . It now becomes clear that this compatibility can only be attained if at least one factor endowment adapts passively to the other variables. This is obviously absurd, however, since factor endowments should all be treated in the Neo-Classical

equilibrium as exogenous data inherited from the past. In short, the entire procedure moves somehow backward. Hahn is so set on encompassing the Sraffian analysis within Neo-Classical theory that he gets himself into the paradoxical position of reversing the temporal axis so that the past is determined as a function of the future. It should be noted, however, that this does not mean that the rate of profit is independent of the technique of production and endowments. As Hahn himself admits, it indicates that what he describes the Sraffian “special case” will only make sense in a wholly fortuitous set of circumstances where the past “history” of the system turns out by chance to be compatible with an exogenously determined rate of accumulation.²⁰

Hahn’s Neo-Classical reinterpretation of the Surplus approach produces a grotesque result. His stubborn insistence on attempting to enclose the Surplus approach within an orthodox model leads him to accuse the Sraffian economists of devoting their energies to incomprehensible exercises based on reversal of the time axis, an interpretation that is bizarre to say the least. As we have seen, however, the reality is that the theorists of the Surplus approach do not regard the distributive variables as depending on the endowments of production factors and do not assign them the function of balancing demand with these exogenous endowments. Their idea is rather that the distribution of income can be determined prior to and independently of production inputs, since these are not given but can in fact adapt to it. Furthermore, it is worth pointing out that the paradoxical character of Hahn’s demonstration shows that it is impossible for Neo-Classical theory to incorporate not only the Sraffian conception of distribution but also the Keynesian analysis of the level of income. In actual fact, Hahn’s “special case” proves also wholly misleading with respect to the interpretation of any Keynesian link between investments and savings. From a Keynesian viewpoint, the exogenous character of the rate of accumulation g indicates that demand determines income and saving as well as the rate of profit, given the Classical saving hypothesis. When Keynes and the Keynesians maintain that investment determines saving, they are therefore saying that demand generates the ensuing income. Hahn’s “special case” describes a Neo-Classical model, however, where the assumption that the rate of accumulation is exogenous leads to a completely different and paradoxical situation in which subsequent demand “constrains” previous saving.²¹

What Hahn seems not able to accept is the fact that in a Surplus approach scheme - based on a Sraffian price analysis and a Keynesian macroeconomic equilibrium - the endowments of production inputs are not given but adapt to the level and composition of effective demand. Labour can be considered abundant, the means of production are themselves produced and their degree of utilization is flexible. So they cannot constitute a constraint in the determination of relative prices and distribution. But Hahn refuses this reasoning and then proposes an inconsistent view of the theorists of Surplus, according to which they assume a given level of accumulation and then are forced to check whether that level is compatible with the inherited endowments of production inputs. In other words, Hahn interprets the Surplus theorists as if they imposed a specific future and then could only hope that it is consistent with a given past.

7. Conclusions

In the Neo-Classical framework, prices and distributive variables are indicators of the scarcity of goods and factors of production in relation to demand. Regardless of the particular focus – the quantity of labour in Pigou, capital expressed as value in Wicksell, physical endowments of goods and factors of production in Hicks's models of temporary equilibrium or Arrow and Debreu's models of intertemporal equilibrium, factor endowments in Solow's analysis or the most recent models of the Neo-Classical theory of growth – these magnitudes are regarded as exogenous. Whether they then also prove binding, and hence scarce, will depend upon whether there is a tendency toward their full utilisation. It thus becomes a fundamental problem of the Neo-Classical models to demonstrate that, under given hypotheses, the mechanism of price formation leads to the full utilisation of existing endowments, i.e. the complete adjustment of demand to endowments. This mechanism should ultimately lead the economic system to a situation of full utilisation equilibrium in which demand corresponds perfectly to the given supply, production is determined by the optimal utilisation of factors, and prices and distributive variables reflect the scarcity of endowments with respect to demand.

The theorists of the Surplus approach have not only helped to show that the mechanism of equilibrium described here does not work in Neo-Classical models with capital²² but also developed a radically alternative view of prices and distribution. Prices and distributive variables are in fact not seen in Surplus-approach analyses as indicators of comparative scarcity because they no longer perform the function of bringing demand into line with endowments of factors of production. As we have seen, this approach regards production inputs as having a variable degree of utilisation, as being reproducible or abundant, and therefore as tending to adapt to the techniques of production chosen on the basis of the exogenous distributive variable. This does not mean, of course, that the Surplus approach completely rules out the existence of quantitative constraints inside the economic system. The possible impact of these constraints on prices and distribution is, however, examined in terms that are completely different from Neo-Classical theory. Consider the possibility of determining the rate of profit on the basis of the equation of macroeconomic equilibrium, for example. As pointed out above, some advocates of the Surplus approach do in fact admit this way of determining profit, the logic of which rests on the idea that an excess of investments over savings could lead to an inflationary trend. On the assumption that monetary wages are comparatively rigid, this means a distributive effect favourable to profits and, on the Classical hypothesis, also an increase in savings such as to restore the macroeconomic equilibrium. This process does affect distribution, but in a way that has nothing to do with the Neo-Classical

mechanism for the determination of distributive variables on the basis of the meeting between demand and exogenous factor endowments. The determination of profit on the basis of macroeconomic equilibrium instead suggests a fundamental asymmetry of power between social classes, since the adjustment is based on the idea that firms can adapt their spending decisions continuously to variations in price, whereas workers tend to be delayed by the comparative rigidity of the monetary wages. It should also be borne in mind that this possible mechanism of adjustment based on the movement of prices is perfectly compatible with a situation of worker unemployment. In this type of analysis, the macroeconomic equilibrium poses a problem of proportions between the variables involved quite independently of the absolute scale of activity and hence employment. This constitutes a further distinguishing characteristic with respect to a Neo-Classical theory that admits no equilibriums characterized by under-utilisation of productive resources in its general models and models of perfect competition.²³ The Surplus approach thus appears to refer in every respect to a specific object of study comprising the capitalist form of production, the under-utilisation of resources that often distinguishes it, and the antagonistic relations between classes developed within it. The Neo-Classical theory instead appears intent on applying the neutral and efficient perspective of scarcity to all the objects of its analysis, regardless of whether it is addressing a Robinson Crusoe economy or contemporary capitalism.

In conclusion, the two approaches are irreducibly antagonistic and it is therefore no wonder that Hahn's attempt to portray the Surplus approach as no more than a "special case" of the Neo-Classical analysis should lead to results of no importance whatsoever for our understanding of the real world. It is to be hoped that the particular interpretation of Hahn within a Solowian framework put forward here will help to make the criticisms of his attempt more widely understood and accepted, not least by economists of the orthodox persuasion.

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¹ I am grateful to Roberto Ciccone and Ian Steedman for their helpful comments on a previous version of this paper. The usual disclaimers apply.

² Some economists have used the term "Neo-Ricardian" in a negative sense to call the connections between this approach and the work of Marx into question. See for example Rowthorn (1974). In our view, the well-known differences regarding the labour theory of value are not sufficient to deny the Marxian legacy of the Surplus approach. Among the authors maintaining the Marxian ancestry of this approach, see Garegnani (1984). About the link between Marx and the Sraffian approach, see also Steedman (1977). For a reconstruction, see Kurz and Salvadori (2008),

Aspramourgos (2004).

³ For one thing, the mathematical system adopted will describe an economy that produces only one good and thus precludes any examination of Sraffa's important criticism of the Neo-Classical theory of capital. For another, it envisages a continuous and differentiable production function with perfect substitutability of the factors of production, which has always been considered wholly unrealistic by Sraffian theorists. Moreover, it assumes constant returns to scale, which Sraffa specifically indicated as unnecessary for the purposes of his analysis.

⁴ Dobb (1973).

⁵ For the influence of Hahn's article in the theoretical debate, see Harcourt (1990).

⁶ See Pasinetti (2000) and Petri (2004).

⁷ Some of these concepts had already been expressed by Hahn (1975). See Bliss (1975) for a similar approach.

⁸ See Mandler (1999) for a recent example.

⁹ Hahn (2003).

¹⁰ For these and other criticisms, see Dumenil e Levy (1985), Schefold (1985), Garegnani (1990), Kurz e Salvadori (1995), Pasinetti (2000), Garegnani (2003), Petri (2003).

¹¹ Garegnani (2003, p. 150).

¹² The point of reference is Hahn's equation (3.18) (1982, section 5).

¹³ This theoretical structure coincides at the conceptual level with the one represented by Hahn's equations (3.17), (3.22), (3.25). See Hahn (1982, section 5).

¹⁴ Some clarification is called for as regards the connection between "non-stationary" and "short-period" equilibrium, which involves a logical link between Solow's analysis and the modern analyses of intertemporal and temporary equilibrium. It should be recalled Solow subjects the behaviour of agents to marked degree of simplification so as to eliminate the problems regarding optimal intertemporal allocation and expectations. The temporal structure of the model is, however, analogous to its more complex counterpart in the modern analyses of Neo-Classical general equilibrium, and it is for this reason that we have been able to incorporate the operation carried out by Hahn (1982) within it. At the same time, it should be specified that the analogy between "short-period" equilibrium and non-stationary Solowian equilibrium is permissible because the model presented here has only one good and therefore treats capital as a homogeneous physical magnitude. If capital were instead expressed in terms of value, a point of non-stationary equilibrium with K given and a unique and uniform rate of profit would have to be regarded as a long-period equilibrium in the classical and traditional Neo-Classical sense of the term (Garegnani 1979). The equilibrium of stationary growth of the Solowian model would instead correspond in that case to an equilibrium described by classical and traditional Neo-Classical economists as "secular". See Garegnani (1976) and Petri (1999). It should of course be borne in mind that with K expressed in terms of value, Solow's model would be subject to the Sraffian criticisms of the Neo-Classical theory of capital and hence prove wholly inconsistent.

¹⁵ For a closer examination, see Brancaccio (2009).

¹⁶ Hahn and Matthews (1964).

¹⁷ In formal terms, it is unquestionably possible to combine a Sraffian system of production prices with an equation of macroeconomic equilibrium. In particular, given the classical saving hypothesis and the assumption that the rate of growth is exogenous, it is possible to obtain the rate of profit from the macroeconomic equilibrium. Given the latter, it is then possible to arrive through the system of production prices at the determination of wages and the relations of exchange between goods. This solution has, however, been put forward only by some of Sraffa's successors, such as Pasinetti and Salvadori, and Pasinetti himself points out that it constitutes only one of the various possible formal closures of a Sraffian system (Pasinetti 1990). Others have instead subjected this procedure to marked criticism based on the idea that Sraffa's exogenous distributive variable – which can be the rate of profit or wages – refers to a "normal" distribution in the classical sense, which is assumed to be characterised by a certain degree of "persistence" and cannot therefore be regarded as directly dependent on the continuous change of macroeconomic variables. Critics have also pointed out that this procedure unduly restricts dynamic analysis of the quantities produced, confining it exclusively to the case of stationary

growth. They have therefore indicated alternative ways of restoring macroeconomic equilibrium that are based no longer on variations in the rate of profit but rather on change in the degree of utilisation of productive capacity or the amount of autonomous expenditure that does not generate additional capacity. See Brancaccio (2003) for an overview and Brancaccio (2008) for an analytical synthesis of these positions.

¹⁸ The procedure that Hahn (1982) encloses in the system of equations from (3.22') to (3.30) is thus presented within the framework of a model with a single good.

¹⁹ In actual fact, Hahn (1982, section 5) uses this expression with reference to the problem of the uniformity of rates of profit rather than the compatibility between the rate of accumulation, rate of profit and optimal ratio of physical capital to labour. The terms of the problem are, however, completely equivalent at the conceptual level in the sense that Hahn's expression refers in any case to what he regards as the primary limitation of the Sraffian Surplus analysis, namely an unavoidable – except by chance – incompatibility between the endowments and the other exogenous variables of the model.

²⁰ Hahn (1982, section 5).

²¹ Hahn seems to realise the problem when he states that «there is an interpretation of g connected with the question of “animal spirits” that could cause the Neo-Classical theory some difficulties». The question is then hastily dismissed, however, with a remark to the effect that «problems of interpretation have no importance in the case considered» (Hahn 1982, section 5). This is, however, somewhat perplexing, as Hahn's article is wholly concerned with putting forward a Neo-Classical interpretation of the Surplus approach.

²² See Petri (2004) for a reconstruction of the debate.

²³ This difference between the two approaches can also be shown within the framework of the Solowian system used here. Consider a case in which the technique k is fixed and exogenous. For the Surplus approach, this hypothesis is in no way detrimental to the solution of the system. Prices and distributive variables can be determined in any case and production inputs will adapt to the only technique available (there is of course nothing in the framework of the Surplus approach to guarantee that all the inputs will be fully utilised). For the Neo-Classical theory, on the contrary, regarding k as exogenous makes it impossible to arrive at an equilibrium distribution and prices. In point of fact, except by chance, k will not coincide with the exogenous endowments of K and L , which will give rise to a permanent imbalance between the firms' demand for factors of production and the available supply of the same. We know, however, that the task performed by prices and distributive variables in the Neo-Classical framework is precisely to bring demand into equilibrium with the given supply. In this case, however, k is fixed and the firms' demand cannot therefore adapt to endowments. It is thus impossible to restore equilibrium and the prices and distributive variables cannot be determined.